Exercise 57

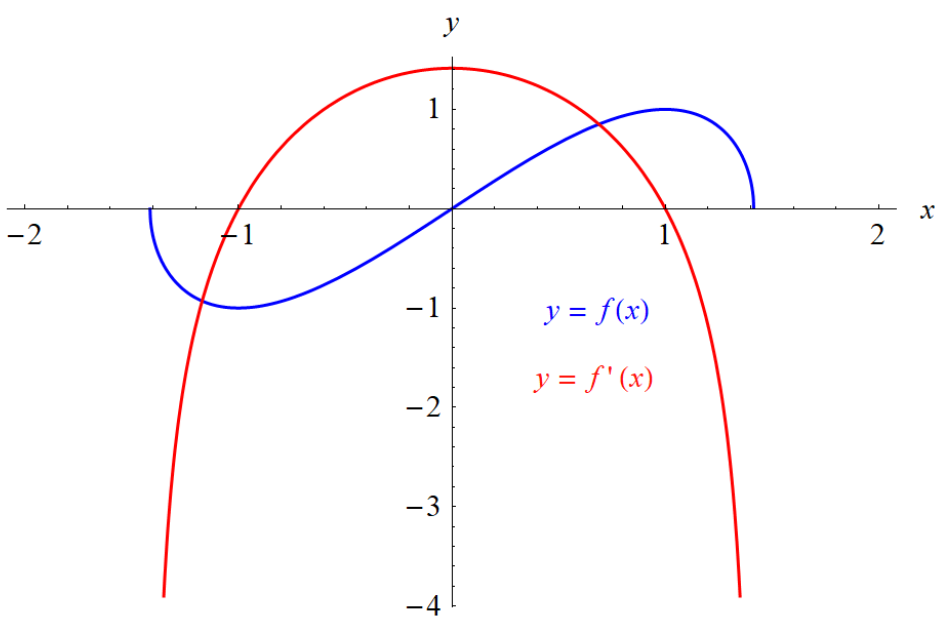
(a) If $f(x) = x\sqrt{2-x^2}$, find $f'(x)$.

(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

Solution

Take a derivative of the given function.

$$\begin{aligned}
f'(x) = \frac{df}{dx} &= \frac{d}{dx}\left(x\sqrt{2-x^2}\right) \\
&= \left[\frac{d}{dx}(x)\right]\sqrt{2-x^2} + x\left[\frac{d}{dx}(\sqrt{2-x^2})\right] \\
&= (1)\sqrt{2-x^2} + x\left[\frac{1}{2}(2-x^2)^{-1/2}\cdot\frac{d}{dx}(2-x^2)\right] \\
&= (1)\sqrt{2-x^2} + x\left[\frac{1}{2}(2-x^2)^{-1/2}\cdot(-2x)\right] \\
&= \sqrt{2-x^2} - \frac{x^2}{\sqrt{2-x^2}} \\
&= \frac{2-x^2}{\sqrt{2-x^2}} - \frac{x^2}{\sqrt{2-x^2}} \\
&= \frac{2(1-x^2)}{\sqrt{2-x^2}}
\end{aligned}$$



The graph of $f'(x)$ is negative wherever $f(x)$ is decreasing, and the graph of $f'(x)$ is positive wherever $f(x)$ is increasing.